MA 3046 - Matrix Analysis

Problem Set 8 - Section VI - Iterative Methods (Partial Set)

1. Consider the system of equations:

$$\begin{array}{rcrcrcr} 9x_1 & + & 2x_2 & - & x_3 & = & -2 \\ -x_1 & + & 6x_2 & - & x_3 & = & 2 \\ 2x_1 & - & x_2 & + & 8x_3 & = & 3 \end{array}$$

Solve this system to four-digit accuracy using the Jacobi algorithm.

2. Resolve problem 1, again to four-digit accuracy, using the Gauss-Seidel algorithm.

3. Consider the system of equations:

$$\begin{array}{rcrcrcr} 10x_1 & - & 2x_2 & + & x_3 & = & 1 \\ 2x_1 & - & 8x_2 & + & x_3 & = & 1 \\ x_1 & - & x_2 & + & 8x_3 & = & -3 \end{array}$$

Solve this system to four-digit accuracy using the Jacobi algorithm.

4. Resolve problem 3, again to four-digit accuracy, using the Gauss-Seidel algorithm.

5. The order in which equations are written can affect whether or not they can be solved, in their original form, by iterative methods. For example, consider

$$\begin{array}{rcrcr} 4x_1 & + & x_2 & = & 1 \\ x_1 & + & 4x_2 & = & 1 \end{array} \quad \text{and} \quad \begin{array}{rcrcr} x_1 & + & 4x_2 & = & 1 \\ 4x_1 & + & x_2 & = & 1 \end{array}$$

Show that the first of these is solvable by either the Jacobi or Gauss-Seidel method, but that not only is the second not solvable by either, but the Gauss-Seidel algorithm diverges faster than Jacobi for that system.

6. As discussed, diagonal dominance is a sufficient, but not necessary condition for convergence of an iterative method. Show that both the Jacobi and Gauss-Seidel methods converge for the system:

$$\begin{array}{rcrcr} x_1 & + & 2x_2 & = & 1 \\ x_1 & + & 10x_2 & = & -2 \end{array}$$

even though the system is clearly not diagonally dominant.